Efficient Band Occupancy and Modulation Parameter Detection

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Abstract

Consider a communication scenario where several users share a band of frequencies dynamically. To share the band efficiently, it is desirable to obtain as much information as possible about its use in real time. Of vital importance for most digital communication systems and for the distinction between analog and digital communication signals is the determination of the baud or symbol rate F_B. We propose a frequency domain method that determines the band occupancy and the symbol rates for all signals in a frequency band simultaneously, without the need to first separate individual transmissions in the band. In a second step, specific signals of interest can then be converted back to the time domain with an appropriately lowered sampling rate to determine such parameters as the modulation method and the signal constellation. Overall, the proposed method reduces the computational effort by about an order of magnitude compared to conventional time domain methods for finding F_B.

1. Introduction

The topic of this paper was motivated by one of the hurdles for participation in DARPA's Spectrum Collaboration Challenge (SC2)(DARPA, 2016). The general goal of SC2 is to find ways to share the finite radio frequency (RF) spectrum dynamically and collaboratively among many users. The task of the "RF Environment Understanding" hurdle is to "Develop a classifier that can identify the occupied range and type of six simultaneous non-overlapping signals within a 3 MHz bandwidth channel." In the noiseless case, the power spectral density (PSD) of the signal set might look as shown in Fig. 1.

The PSD in Fig. 1 was obtained from 3 seconds worth of complex-valued data (I and Q samples) sampled at a rate

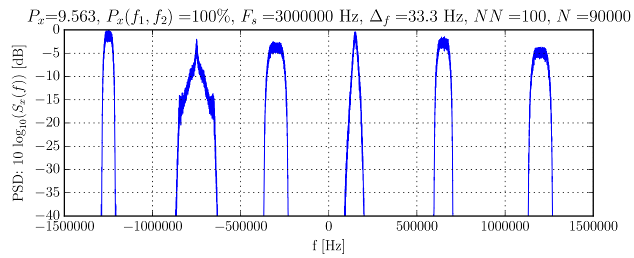

Figure 1. PSD of Noiseless Signals in a 3 MHz Band

of $F_s = 3.0$ MHz. The graphical display is averaged over 100 blocks of length $N = 90000$. Some of the signals in the band are analog and some are digital. The goal is to be able to say, for instance, signal X_0 is a 8-PSK signal with center frequency $f_{c0} = -1200$ kHz and symbol rate $F_{B0} = 15$ kHz, signal X_1 is a 16-QAM signal with $f_{c1} = -650$ kHz and $F_{B1} = 60$ kHz, signal X_2 is a FM signal with $f_{c2} = -80$ kHz, etc. The identification of the signals ideally takes place in real time, so that a user wishing to transmit in the band can identify opportunities to transmit, e.g., because it is known that some of the digital signals are mostly transmitted in bursts. The situation gets a little more complicated when there is noise, as shown in the PSD of Fig. 2 for a signal to noise ratio (SNR) of about 10 dB.

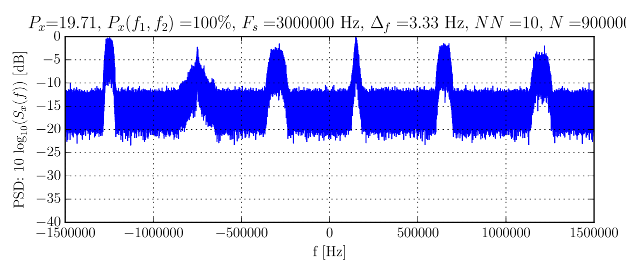

Figure 2. PSD of Signals with SNR approximately 10 dB in a 3 MHz Band

In the following we will use the noiseless signal set for clarity of exposition and the noisy signal set to explore limitations.

2. Conventional Method

The first action item is the determination of the (approximate) locations of the center frequencies and the bandwidths of the active signals. A well known method that can be used is Welch's modified periodogram method (Welch, 1967). For the noiseless case we may see a graph similar to the one shown in Fig. 3 which was created from a discrete Fourier transform (DFT) of length $N = 30000$ (corresponding to 10 ms of data in the time domain).

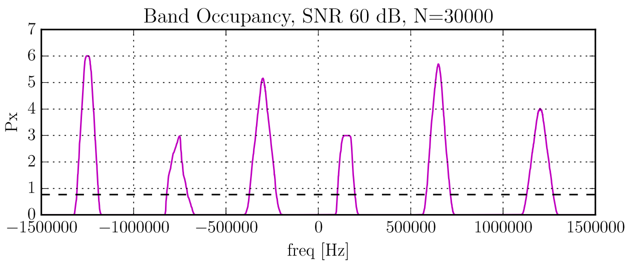

Figure 3. Determination of bands and center frequencies, noiseless case

Labeling the signals from left to right as X_0, X_1, ..., X_5, we obtain the following approximate values for the center frequencies f_c and the bandwidths BW.

Signal	f_c [kHz]	BW [kHz]
X_0	-1250	100
X_1	-750	73
X_2	-300	120
X_3	160	76
X_4	650	105
X_5	1200	120

A well known technique to obtain the symbol rate of a digital signal is to pass the time domain signal through a nonlinearity and then to look at the spectrum of the result, see for instance Chapter 16 in (Barry et al., 2004). Let $x(t)$ denote the signal set consisting of 6 simultaneously transmitted signals. We can write

$$x(t) = \sum_{i=0}^{5} s_i(t)\, e^{j(2\pi f_{ci} t + \theta_i)}, \tag{1}$$

where $s_i(t)$ are the baseband signals, f_{ci} are the carrier frequencies and θ_i are the carrier phases. Taking the PSD of $|x(t)|^2 = x(t)\, x^*(t)$, where * denotes complex conjugate, yields the graph in Fig. 4.

Because the complex-valued signal $x(t)$ is magnitude-squared (and not just squared), the carrier frequency and phase information drops out and Fig. 4 shows the superposition of all baseband signals (magnitude-squared). The

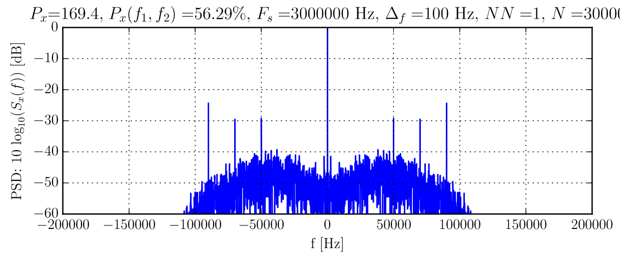

Figure 4. Spectrum of Noiseless Signal $|x(t)|^2$

spectral lines in the graph show the superposition of **all** symbol rates in the signal set $x(t)$ (at 50, 70, and 90 kHz). Thus, this looks like an elegant method to determine all symbol rates of the signals present in a band of frequencies. However, if the signal $x(t)$ is noisy, the situation deteriorates rapidly as shown in Fig. 5 for a SNR of approximately 10 dB.

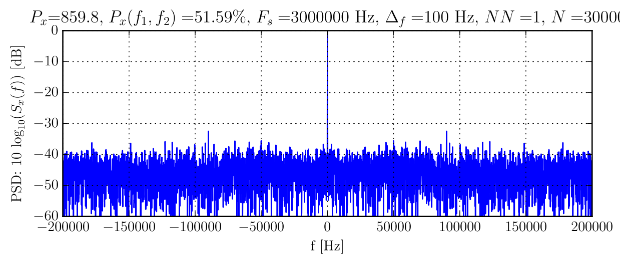

Figure 5. Spectrum of Noisy Signal $|x(t)|^2$, SNR approximately 10 dB

In the realistic case of observing a noisy signal set it is therefore necessary for the "magnitude-squaring followed by PSD" method to filter out each indivdual signal before squaring. Fig. 6 shows the result when one of the noisy signals (X_5, for example) is filtered first (e.g., by multiplying with $\exp(-j2\pi f_{ci} t)$ followed by lowpass filtering) before taking the PSD of the magnitude-squared result.

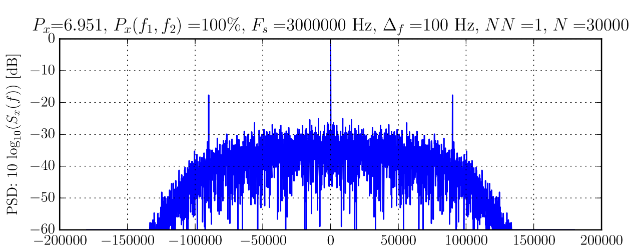

Figure 6. Spectrum of Filtered Noisy Signal X_5 after Magnitude-Squaring, SNR approximately 10 dB

Thus, even in the noisy case, it is possible to obtain the symbol rates of the signals in a frequency band, but it requires to operate on each signal individually.

3. Frequency Domain Method

To avoid the inefficiencies associated with operating on individual signals contained in a band of frequencies, we propose to work with the Fourier transform (FT) $X(f)$ of the composite signal $x(t)$ to obtain the FT of the magnitude-squared signal $|x(t)|^2 = x(t)x^*(t)$ as follows.

$$\begin{aligned}
&\int_{-\infty}^{\infty} x(t)\, x^*(t)\, e^{-j2\pi f t}\, dt = \\
&\quad = \int_{-\infty}^{\infty} x(t) \int_{-\infty}^{\infty} X^*(\nu)\, e^{-j2\pi \nu t}\, d\nu\, e^{-j2\pi f t}\, dt \\
&\quad = \int_{-\infty}^{\infty} X^*(\nu) \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (f+\nu) t}\, dt\, d\nu \\
&\quad = \int_{-\infty}^{\infty} X(f+\nu)\, X^*(\nu)\, d\nu
\end{aligned} \tag{2}$$

Note that, even though we talk about the (continuous time and continuous frequency) FT here, in practice the signal $x(t)$ is sampled with some sampling rate F_s and the FT is approximated for the frequencies of interest using a DFT or FFT (discrete or fast Fourier transform). As can be seen from eq. 2, the magnitude-squaring operation in the time domain corresponds to a (auto)correlation function in the frequency domain. The crucial observation is that for each signal $x_i(t)$ in the signal set $x(t)$ only a finite range of frequencies, where $X_i(f+\nu)$ and $X_i^*(\nu)$ overlap, needs to be included in the correlation integral. This is shown graphically for $f = 50$ kHz in Fig. 7.

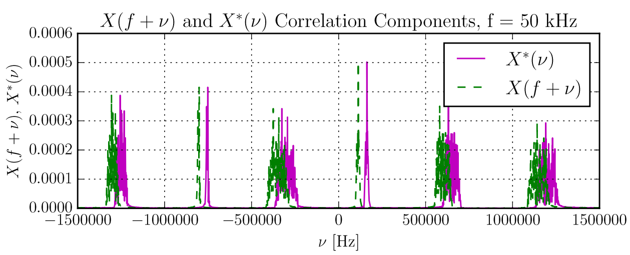

Figure 7. Component Spectra for Frequency Domain Correlation

As f in eq. 2 is increased from 0 to $\max(F_{Bi})$ (the maximum of all symbol rates), the local integrations over the individual signal overlaps should result in local peaks when $f = F_{Bi}$ for the i-th signal. Define

$$C_W(f_x, F_{BT}) = \int_{f_x - F_{BT}/2 - W/2}^{f_x - F_{BT}/2 + W/2} X(F_{BT}+\nu)\, X^*(\nu)\, d\nu\ , \tag{3}$$

where F_{BT} is a specific symbol rate to be tested. The quantity $C_W(f_x, F_{BT})$ is a bandlimited correlation of bandwidth W around center frequency f_x, in the frequency domain with frequency shift F_{BT}. The parameter W can be varied to trade off precision in spectral line location versus robustness to noise. A good starting point for W is the bandwidth of the signal at frequency f_x. If the parameter F_{BT} is set to 0 then $C_W(f_x, 0)$, $-F_s/2 \leq f_x \leq F_s/2$, yields the band occupancy graph shown in Fig. 8 for a noisy signal $x(t)$ and $W = 80$ kHz. The blocklength of the DFT used for the figure is $N = 30000$.

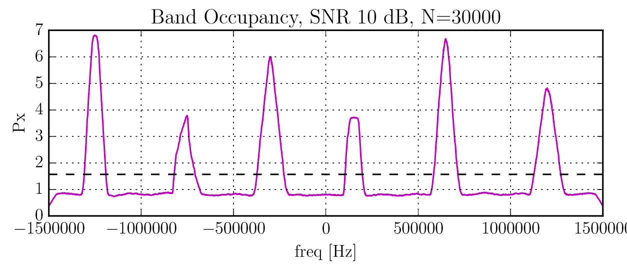

Figure 8. Determination of bands and center frequencies, noisy case

As F_{BT} is increased, a 3-dimensional plot can be obtained with x-axis f_x, y-axis F_{BT}, and z-axis $C_W(f_x, F_{BT})$. For the noiseless signal set, bandwidth $W = 80$ kHz, and DFT blocklength $N = 30000$, the result is shown in Fig. 9.

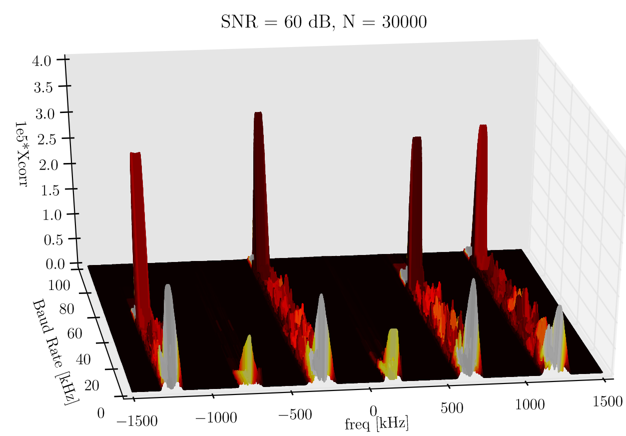

Figure 9. Combined Determination of Bands and Symbol Rates, Noiseless Case

On the x-axis the spectral peaks occur at $f_{ci} - F_{Bi}/2$ and on the y-axis the spectral peaks occur at F_{Bi}. The grey peaks visible at the front of the graph show the band occupancy (scaled back by a factor of 10 to not obscure the rest of the graph). Thus, with one DFT and some bandlimited correlations in the frequency domain, it is possible to obtain the band occupancy as well as the symbol rate data for the digital signals in the signal set. In Fig. 9 it is clearly visible that signals X_0, X_2, X_4 and X_5 (indexing from left, starting with index 0) are digital signals. Signal X_0 has the lowest symbol rate, and signals X_2 and X_5 have both about the same (highest) symbol rate. The symbol rate of X_4 lies roughly halfways in between. It is also easy to see that signals X_1 and X_3 have no spectral peaks beside the band occupancy at $F_{BT} = 0$ and therefore they can be classified either as analog or as (digital) constant envelope signals.

If the blocklength of the initial DFT is chosen large enough, then the frequency domain method with the bandlimited correlations is quite robust in the presence of noise. Fig. 10 shows $C_W(f_x, F_{BT})$ for the noisy signal set with SNR approximately 10 dB and all other parameters set as for Fig. 9.

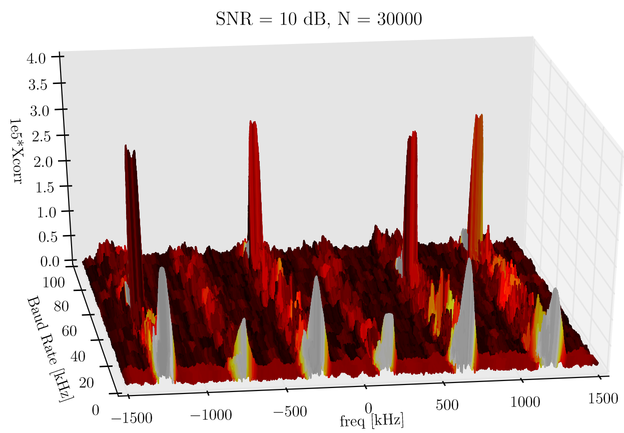

Figure 10. Combined Determination of Bands and Symbol Rates, SNR approximately 10 dB

The added noise is clearly visible in Figs. 8 and 10, but the spectral peaks extend well above the noise floor.

4. More Modulation Parameters

Not all modulation parameters can be determined in the frequency domain. To obtain the constellation of a signal in the IQ (in-phase/quadrature) space, for example, the imaginary (quadrature) part needs to be displayed versus the real (in-phase) part of the time domain signal. If the FT $X(f)$ is already available and the center frequencies f_{ci} and bandwidths BW_i of the individual signals

$$X_i(f) \quad \Longleftrightarrow \quad x_i(t) = s_i(t)\, e^{j2\pi f_{ci} t} \tag{4}$$

in $x(t)$ are known, then a specific $s_i(t)$ can be generated efficiently from $X(f)$ as follows. Select $X_i(f)$ which is equal to $X(f)$ for $f_{ci} - BW_i/2 \leq f < f_{ci} + BW_i/2$ and shift it to baseband to obtain $S_i(f) = X_i(f - f_{ci})$. The result is shown in Fig. 11 for signal X_5 (the far right signal in Fig. 1).

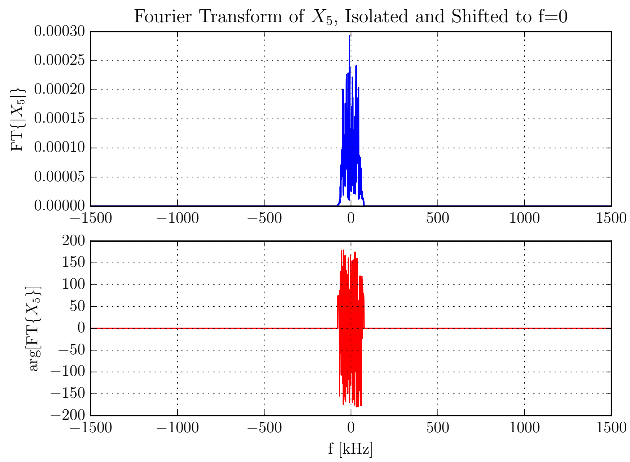

Figure 11. Fourier Transform of Signal X5, Isolated and Shifted to f=0

If the baud rate F_{Bi} of the i-th signal is already known, then there is no need to take an inverse FFT (IFFT) over the full original bandwidth used for the composite signal $x(t)$. Thus, the bandwidth can be reduced, e.g., corresponding to 3.33 samples per symbol in the time domain (equivalent to downsampling by a factor of 10 in the time domain for the parameters used here) as shown for signal X_5 in Fig. 12.

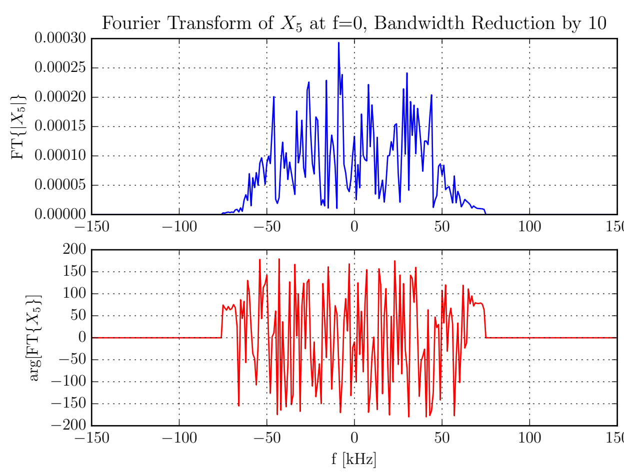

Figure 12. Fourier Transform of Signal X5 at f=0, Bandwidth Reduction by 10

In terms of a DTFT (discrete-time FT), Fig. 11 shows one period of the spectrum of a DT signal sampled at $F_{s1} = 3$ MHz whereas Fig. 12 shows the spectrum of a (downsampled) DT signal at $F_{s2} = 300$ kHz. After conversion back to the time domain, the IQ plot shown in Fig. 13 for signal X_5 with the sampled signal points rendered as red circles results. Note that, even though the figure shows the noiseless case, intersymbol interference (ISI) is vsible because a lowpass filter instead of a matched filter was used to receive the RRCf (root raised cosine in frequency) signal.

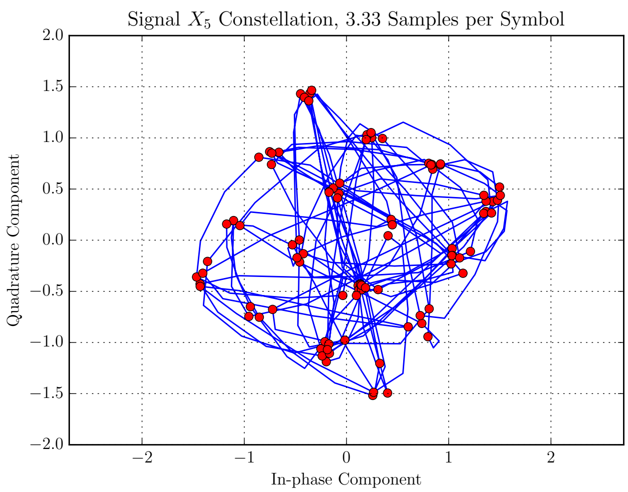

Figure 13. Signal Constellation of noiseless X5, 3.33 Samples per Symbol

But it is clearly visible that signal X_5 is a 16-QAM signal and that remains true even if the signal is more noisy, as depicted in Fig. 14 with an SNR of about 20 dB.

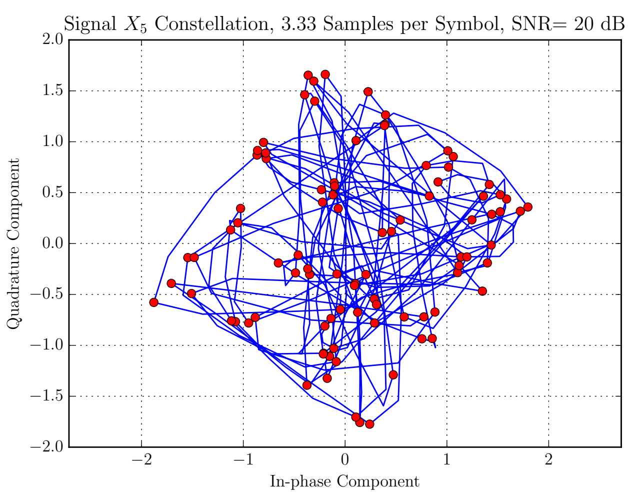

Figure 14. Signal Constellation of noisy X5, 3.33 Samples per Symbol, 20 dB SNR

5. Computational Effort Comparison

Let F_s be the sampling rate of the composite complex-valued (after IQ downconversion) signal $x(t)$ and let Δf be the desired frequency resolution for the baud rate determination. Then a blocklength of $N \geq F_s/\Delta f$ is needed for conversion between time and frequency domains using a DFT or FFT. Also, let BW denote the "typical" bandwidth of the individual signals contained in $x(t)$. In our example computations we used $F_s = 3$ MHz, $\Delta f = 100$ Hz, and $N = 30000$.

For both, the conventional and the frequency domain methods, we assume that an initial DFT/FFT of length N is performed to determine the approximate center frequencies f_{ci} and the approximate bandwidths BW_i of the individual signals in $x(t)$. The computational effort, measured in multiply-accumulate (MAC) instructions or units, for this step is about $N \log_2 N$ and is roughly the same for both methods.

The next steps for the conventional method are to (*numbers in parentheses are for the example computations*)

- shift each individual signal to baseband by computing $x(t)e^{-j2\pi f_{ci}t}$
- lowpass filter the result for each individual signal with a filter of bandwidth $BW/2$ to obtain $s_i(t)$, requiring a (FIR) filter of order n approximately equal to $2N\Delta f/(BW/2)$. This uses about $4N^2\Delta f/BW$ MACs. (*n=120, 3.6e6 MACs*)
- square each $s_i(t)$ and compute the DFT/FFT of the result, using about $N \log_2 N$ MACs (*4.5e5 MACs*)

For the six signals in the example file used here, the grand total to obtain all baud rates (or their absence) using the conventional method is on the order of *2.5e7 MACs*.

For the frequency domain method eq. 3 needs to be evaluated for each $f_x = f_{ci}$ and for each trial baud rate F_{BT}. A good rule of thumb for W in eq. 3 is to use $W = BW$. From the examination of the initial DFT/FFT, the range of values for f_x and F_{BT} that need to be evaluated for finding the actual F_{Bi} (or recognizing their absence) can be significantly reduced. Note that RRCf based signals will have $F_{Bi} \approx BW_i$. Under the above assumptions, each (discrete frequency) evaluation of eq. 3 requires $BW/\Delta f$ MACs. Letting $f_x = f_{ci} \pm 0.1\,BW$ and $F_{BT} = BW_i \pm 0.1\,BW$, eq. 3 needs to be evaluated $1.2\,BW/(5\Delta f)$ times for each $x_i(t)$, thus requiring $0.24\,(BW/\Delta f)^2$ MACs. Note that the factor of 1.2 results from extending the range of f_x in the integration limit by $\pm 10\%$. For the numerical example values used here, this results in a computational effort of *1.44e6 MACs* for obtaining all baud rates (or their absence) when the frequency domain method is used. This is an improvement by a factor of almost 20 when compared to the conventional method.

6. Limitations

The conventional as well as the frequency domain methods do require about one hundred or more data symbols and enough frequency resolution to accurately determine the symbol rate of a digital communication signal. To analyze a whole band of, say 10 (non-overlapping) signals, at least $2 \times 10 = 20$ samples per symbol are required. For a 100 Hz resolution, a time segment of length 10 ms is needed. The latter constraint is typically more dominant, resulting in the computation of FFTs of blocklengths $N \approx 0.01\, F_s$ (*N=30000* for $F_s = 3$ MHz).

One important conceptual difference between the conventional and the frequency domain method is that the former produces a spectral line at the actual symbol rate F_{Bi}, whereas the latter produces a spectral line at $f_{ci} + F_{BT}/2$ only if the trial symbol rate F_{BT} is close enough to the actual rate F_{Bi}.

Constant envelope modulation schemes, such as CPM (continuous phase modulation), CPFSK (continuous phase frequency shift keying), or GMSK (Gaussian minimum shift keying) produce complex-valued baseband signals of the form

$$s(t) = A\, e^{j(2\pi f_c t + \phi(t))}\,, \tag{5}$$

where A is some constant amplitude and $\phi(t)$ is either directly a waveform containing the (digital) data to be transmitted (for phase modulation), or an integral of such a waveform (for frequency modulation). Clearly, taking the magnitude squared of such a signal will be equal to A^2 which does not contain any timing information.

To illustrate this case, we look at the “samples_011” signals (generated at random according to DARPA’s hurdle2 for SC2) that consist of a mix of 6 FM, QPSK and GMSK signals at a SNR of approximately 20 dB. The PSD of this signal set is shown in Fig. 15.

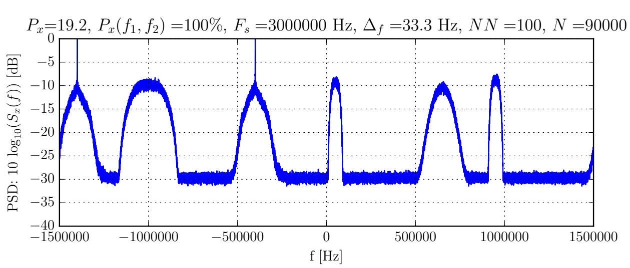

Figure 15. PSD of ”samples_011” Signals, 20 dB SNR

Labeling the signals from left to right as $X11_0, X11_1, \ldots, X11_5$, we find that

Signal	f_c [kHz]	BW [kHz]
$X11_0$	-1400	100
$X11_1$	-1000	230
$X11_2$	-400	100
$X11_3$	50	70
$X11_4$	650	200
$X11_5$	950	70

Using the frequency domain method of Section 3 yields the 3D Occupancy/Symbol Rate graph shown in Fig. 16.

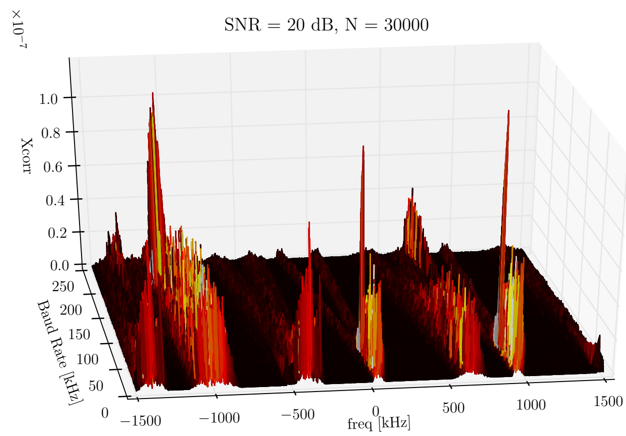

Figure 16. Band Occupancy and Symbol Rates for ”samples_011” Signals, SNR approximately 20 dB

Signals $X11_1, X11_3, X11_5$ have spectral lines at $200, 50, 50$ kbaud, respectively (all three are QPSK signals). But signals $X11_0, X11_2, X11,4$ have no distinctive spectral lines that could be interpreted as symbol rates. From the PSD in Fig. 15 we suspect that $X11_0$ and $X11_2$ are analog FM signals (because of the spectral line at f_c that comes from silence or pauses in the analog modulating signal). To obtain more information about $X11_4$, the signal at $f_{c4} = 650$ kHz is filtered out and shifted to baseband (all operations are directly performed on the intital FT) and then transformed back to the time domain to obtain the complex baseband signal $s_4(t)$. To obtain the symbol rate of this (still constant envelope) signal, compute $[\text{Re}\{s_4(t)\}]^2$, where Re$\{.\}$ denotes taking the real part, and then look at the PSD of the result which is shown in Fig. 17.

Now it can be seen that $F_{B4} = 107.14$ kbaud. To verify that this is indeed correct and that the signal is a GMSK signal, plot the imaginary part of $s_4(t)$ versus its real part. This is shown in Fig. 18 with the actual sampled symbols plotted as red circles.

Thus, signal $X11_4$ is confirmed as a GMSK signal with $F_{B4} = 107.14$ kbaud.

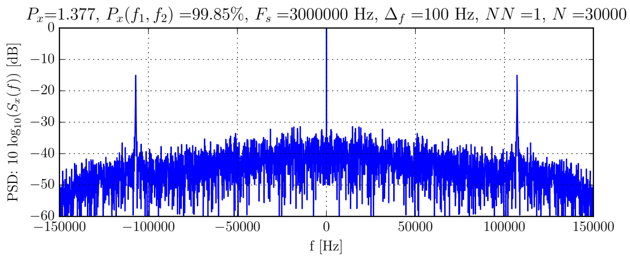

Figure 17. Peaks at Symbol Rate for GMSK Signal X4 from "samples_011" Signals, 20 dB SNR

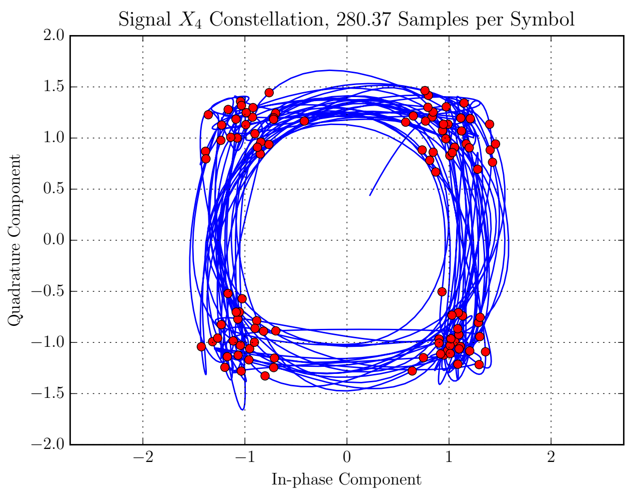

Figure 18. Signal Constellation for GMSK Signal X4 from "samples_011" Signals, 20 dB SNR

7. Python Code

The Python code below shows how to generate the 3D plot shown in Fig. 9 using the frequency domain method. To keep the code simple, it has not been optimized for speed and/or computational efficieny. The data files and a Jupyter notebook can be found on GitHub at https://github.com/mathys2000/..
..BandOccupancyAndModulationDetection

```
from pylab import *
%matplotlib notebook
rc('text', usetex=True)
from mpl_toolkits.mplot3d import Axes3D
rt = fromfile('mysamples03_SNR60dB.dat',\
     dtype=complex64,count=-1)
SNR = 60
L = length(rt)
Fs = 3000000          # Sampling rate
tt = arange(L)/float(Fs)    # Time axis
deltaf = 100          # Frequency resolution
FBTmax = 100000       # Max trial baud rate
# Select short signal segment
x0t0 = 1.0            # Start time
x0tlen = 1/float(deltaf)    # Duration
ixx0 = where(logical_and(tt>=x0t0,\
            tt<x0t0+x0tlen))[0]
N0 = len(ixx0)        # Blocklength
x0t = rt[ixx0]        # Signal segment
tt0 = arange(N0)/float(Fs) # t-axis for x0t
# Compute "FT" of x0t
X0f = fft(x0t)/float(Fs)    # FT for x0t
XX0f = hstack((X0f,X0f))    # Extnd -Fs...Fs
fff0 = (Fs/float(N0))*arange(-N0,N0)
ixf0 = where(logical_and(fff0>=-Fs/2.0,\
            fff0<Fs/2.0))[0]
X0f = XX0f[ixf0]      # FT for -Fs/2...Fs/2
ff0 = fff0[ixf0]
# (Auto-) Correlation in frequency domain
w0 = 80000  # Freq domain window in Hz
h0f = ones(w0/float(deltaf))
f0corr = 0        # Offset 0 correlation
ixf0corr = where(logical_and(\
fff0>=f0corr-Fs/2.0,fff0<f0corr+Fs/2.0))[0]
X0corr = XX0f[ixf0corr]*conj(X0f)
X0corrBF = [0.1*abs(convolve(X0corr,h0f,\
                    'same'))]
FBTs = arange(0,FBTmax,1000) # Trial FB's
for f0corr in FBTs[1:]:
    ixf0corr = where(logical_and(\
               fff0>=f0corr-Fs/2.0,\
               fff0<f0corr+Fs/2.0))[0]
    X0corr = XX0f[ixf0corr]*conj(X0f)
    X0corrBF = vstack((X0corrBF,\
        abs(convolve(X0corr,h0f,'same'))))
mx = amax(X0corrBF.flatten())
x = ff0    # Frequency axis
y = FBTs   # Symbol rate axis
X, Y = meshgrid(x, y)
my_col = cm.hot(X0corrBF/(0.35*mx))
f3 = figure(figsize=(12,6))
af31 = f3.add_subplot(111,projection='3d')
af31.plot_surface(X/1000, Y/1000,\
            X0corrBF, facecolors = my_col)
af31.set_xlabel('freq [kHz]')
af31.set_xlim(-1500,1500)
af31.set_ylabel('Baud Rate [kHz]')
af31.set_ylim(0,FBTmax/1000)
af31.set_zlim(0,mx)
af31.ticklabel_format(style='sci',\
                axis='z', scilimits=(0,0))
af31.set_zlabel('Xcorr')
af31.set_title('SNR = {:d} dB, N = {:d},\
                  $\Delta f$ = {:d} Hz'.\
                  format(SNR,N0,deltaf))
af31.view_init(30, 262)
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References

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